

Various representations of infrared effective lattice QCD

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We study various representations of the infrared effective theory of SU(2) gluodynamics starting from the monopole action derived recently. We determine the coupling constants in the abelian-Higgs model directly from lattice QCD and evaluate the type of the QCD vacuum. The string action is derived using the BKT transformation on the lattice. At the classical level this action reproduces the physical string tension with a good accuracy.

1. INTRODUCTION

The infrared effective theory of QCD is important for the analytical understanding of hadron physics. Abelian monopoles which appear after abelian projection of QCD [1] seem to be relevant dynamical degrees of freedom for infrared region [2]. Shiba and Suzuki [3] derived the monopole action from vacuum configurations obtained in Monte-Carlo simulations extending the method developed by Svendsen. The monopole action was a quadratic form of monopole currents, and was not discrete rotational invariant.

In Section 2, we present new results of a discrete rotational invariant monopole action, the renormalized couplings are determined by the Svendsen method, the monopoles being extracted in the maximal abelian projection. From this monopole action we derive the dual abelian-Higgs model (Section 3) and the string model (Section 4) as an effective theory for SU(2) gluodynamics.

2. MONOPOLE ACTION

We study a discrete rotational invariant action which includes 2-, 4- and 6-point interactions:

$$\mathcal{Z}^{mon} = \sum_{\substack{*k \in \mathbb{Z} \\ \delta *k = 0}} \exp \{-S^{mon}\}$$

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$$\begin{aligned} S^{mon}[*k] &= \frac{g_m^2(b)}{2} (*k, \Delta^{-1} *k) + p(b) \|*k\|^2 \\ &+ q(b) \sum_x \left(\sum_{\mu=-D}^D *k_{x,\mu}^2 \right)^2 \\ &+ r(b) \sum_x \sum_{\mu,\nu,\delta=-D}^D *k_{x,\mu}^2 *k_{x,\nu}^2 (3 *k_{x,\delta}^2 + *k_{x+\nu,\delta}^2) \\ &+ \sum_i f_i S_2^{(i)}[*k], \end{aligned}$$

the closed monopole currents $*k$ are defined on the dual lattice, Δ^{-1} is the lattice Coulomb propagator, $S_2^{(i)}[k]$'s are additional quadratic forms of monopole currents which are introduced to check whether there are any corrections to the Coulomb interaction. It occurs that these corrections are negligibly small for the infrared region, $b(\beta, n) = na(\beta) = \sqrt{\tilde{\kappa}(\beta, n)/\kappa_{phys}}$ is the physical length in unit of the physical string tension κ_{phys} . The dimensionless string tension $\tilde{\kappa}$ is determined by the lattice Monte-Carlo simulation, $a(\beta)$ is the lattice spacing and n is the monopole extension.

We determine the coefficients $g_m^2(b)/2$, $p(b)$, $q(b)$, $r(b)$ numerically using the extended Svendsen method [4]. It turns out that these couplings depend only on b and the scaled action almost lies on the renormalized trajectory. The best fit for the renormalized couplings gives:

$$p(b) = \frac{0.53(1)}{b^{3.56(4)}},$$

$$\begin{aligned}
\frac{g_m^2(b)}{2} &= 0.26(3) \left(\frac{4\pi}{g(b)} \right)^2, \\
q(b) &= \frac{.018(5)}{b^{6.08(4)}}, \\
r(b) &= \frac{.179(14) \times 10^{-3}}{b^{11.40(16)}},
\end{aligned}$$

where

$$\begin{aligned}
g^{-2}(b) &= \frac{11}{24\pi^2} \ln \left(1 + \frac{1}{(b\Lambda)^2} \right) \\
&+ \frac{17}{44\pi^2} \ln \left[1 + \ln \left(1 + \frac{1}{(b\Lambda)^2} \right) \right], \quad \Lambda = 0.75(5).
\end{aligned}$$

The dependence of g on b is consistent with the 2-loop running coupling for the small b region and reproduces the experimental power behavior for the large b region.

3. DUAL ABELIAN- HIGGS MODEL FROM MONOPOLE ACTION

The partition function of the dual abelian-Higgs model is

$$\begin{aligned}
\mathcal{Z}^{A.H.} &= \int_{-\infty}^{+\infty} \mathcal{D}\theta \int_{-\infty}^{+\infty} \mathcal{D}\phi \exp \{ -\mathcal{S}^{A.H.} \}, \\
\mathcal{S}^{A.H.}[\theta, \phi] &= \sum_P \frac{\beta}{2} \theta_P^2 \\
&+ \lambda \sum_n (\phi^*(n)\phi(n) - 1)^2 + \sum_n \phi^*(n)\phi(n) \\
&- \gamma \sum_{n,\mu} (\phi^*(n)U_\mu(n)\phi(n+\mu) + h.c.),
\end{aligned}$$

where $U_\mu = \exp(i\theta_\mu)$ is the dual gauge field, θ_P is the field strength tensor and $\phi(n) = \rho_n \exp(i\varphi_n)$ is the complex Higgs field.

One can rewrite the above integral as the sum over the closed monopole currents $*k$ using the analogue of the BKT transformation [5]. The monopole action calculated in the saddle-point approximation up to $O(\lambda^{-2})$ terms has the form:

$$\begin{aligned}
\mathcal{S}^{\text{mon}}[*k] &= \frac{1}{4\beta} (*k, \Delta^{-1} *k) \\
&+ \left\{ \frac{1}{4\gamma} + \frac{1}{\lambda} \left(\frac{15}{32\gamma} - 1 \right) + \frac{1}{\lambda^2} \left(-\frac{199}{2^6\gamma} - \frac{695}{2^7} + 4\gamma \right) \right\} \|*k\|^2 \\
&+ \left\{ -\frac{1}{2^8\gamma^2\lambda} + \frac{1}{\lambda^2} \left(\frac{71}{2^{10}\gamma} - \frac{157}{2^{12}\gamma^2} \right) \right\} \sum_x \left(\sum_{\mu=-D}^D *k_{x,\mu}^2 \right)^2
\end{aligned}$$

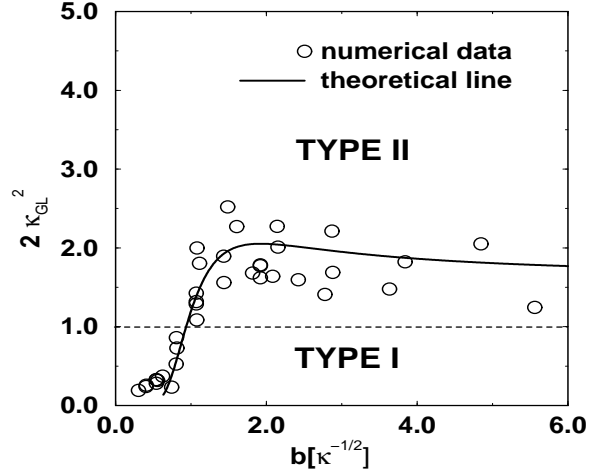


Figure 1. The b dependence of the Ginzburg-Landau parameter for the QCD vacuum. The dotted line denotes critical line between type-I and type-II superconductivity.

$$\begin{aligned}
&- \frac{1}{2^{11}\gamma^2\lambda^2} \sum_x \sum_{\mu=1}^D *k_{x,\mu}^4 + \frac{1}{2^{12}\gamma^3\lambda^2} \sum_x \left(\sum_{\mu=-D}^D *k_{x,\mu}^2 \right)^3 \\
&+ \mathcal{O}\left(\frac{1}{\lambda^3}\right).
\end{aligned}$$

We consider the terms up to $O(\lambda^{-1})$ and we determine the b dependence of the parameters β, γ, λ from the monopole action obtained numerically by the Swendsen method.

One can estimate the type of the superconductivity of the QCD vacuum from the Ginzburg-Landau parameter κ_{GL} defined by

$$2\kappa_{GL}^2 = \frac{m_H^2}{m_A^2} = \frac{4\beta\lambda}{\gamma^2}.$$

It occurs that the QCD vacuum is a type-II superconductor for $b \geq 1.0 [\kappa^{-1/2}]$ (see Figure 1).

4. LATTICE QCD STRING

Using the transformation suggested in Ref. [6] one can also get the effective QCD string model from the monopole partition function:

$$\begin{aligned}
\mathcal{S}^{str}[\sigma] &= \frac{\pi^2}{p} \left(\sigma, \frac{1}{\Delta + M^2} \sigma \right) \\
&- \frac{\pi^2}{8p\tilde{\lambda}M^2} \left(\sigma, \frac{\Delta(0) + \Delta(1)}{(\Delta + M^2)^2} \sigma \right) + \mathcal{O}\left(\frac{1}{M^4}\right),
\end{aligned}$$

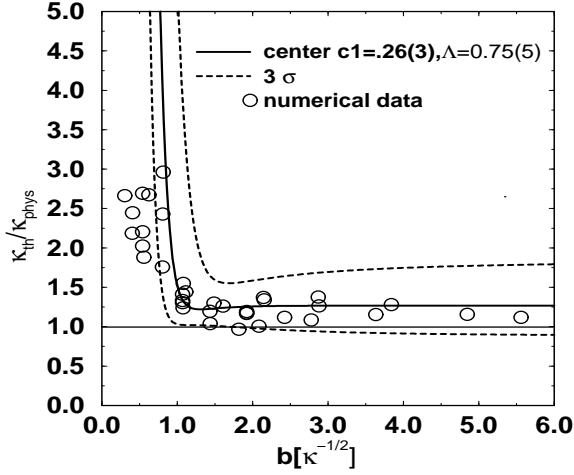


Figure 2. The ratio of theoretical and experimental string tension.

where $M^2 = 1/4p\beta$ and $\tilde{\lambda} = p^2/16q$. The integer-valued 2-form σ which is defined on the original lattice represents the closed world surface formed by a color electric flux tube.

The leading part of this model comes from the self interaction and from the Coulomb interaction of the monopole action; the next-leading terms come from the $*k^4$ - and $*k^6$ -interactions.

One can evaluate the string tension using this string model. The Wilson loop can be written as follows:

$$\langle W \rangle_m = \frac{1}{Z} \sum_{\substack{\sigma(c_2) \in Z \\ \delta\sigma = J}} \exp \{ -S^{str}[\sigma] \},$$

where J is the rectangular source of a quark-antiquark pair.

In the leading order the classical string tension is:

$$\kappa_{th} = \frac{\pi^2}{b^2 p (\Delta + M^2)} (0).$$

We calculate it numerically up to M^{-2} order ($M^2 \gg 1$ in the large b region). For $b \geq 1.0 [\kappa^{-1/2}]$ the theoretical string tension is almost constant and reproduces the physical string tension fairly well (see Figure 2).

5. CONCLUSIONS

- We got the dual Ginzburg-Landau type (abelian-Higgs) theory, its parameters are fixed directly from the lattice QCD. The QCD vacuum is similar to the type-II dual superconductor.
- From the effective abelian-Higgs theory we derive the effective string model in which the classical string tension almost coincides with the physical string tension in the infrared region. The difference between theoretical and experimental string tensions may be due to quantum corrections.
- The obtained QCD string model can be rewritten as follows:

$$\mathcal{S}^{str} = 4\pi^2\beta \|\sigma\|^2 - \frac{4\pi^2\beta}{M^2} (\sigma, \Delta\sigma) + \mathcal{O}\left(\frac{1}{M^4}\right)$$

The leading part of this action corresponds to the Nambu-Goto action in the continuum theory. The non-leading terms probably solve the serious problems of the simple string model such as the critical dimension, because the original QCD has no such problems.

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